A Simultaneous Optimisation of a Leader-follower Train Pair for Safe and Energy-Efficient Train Control

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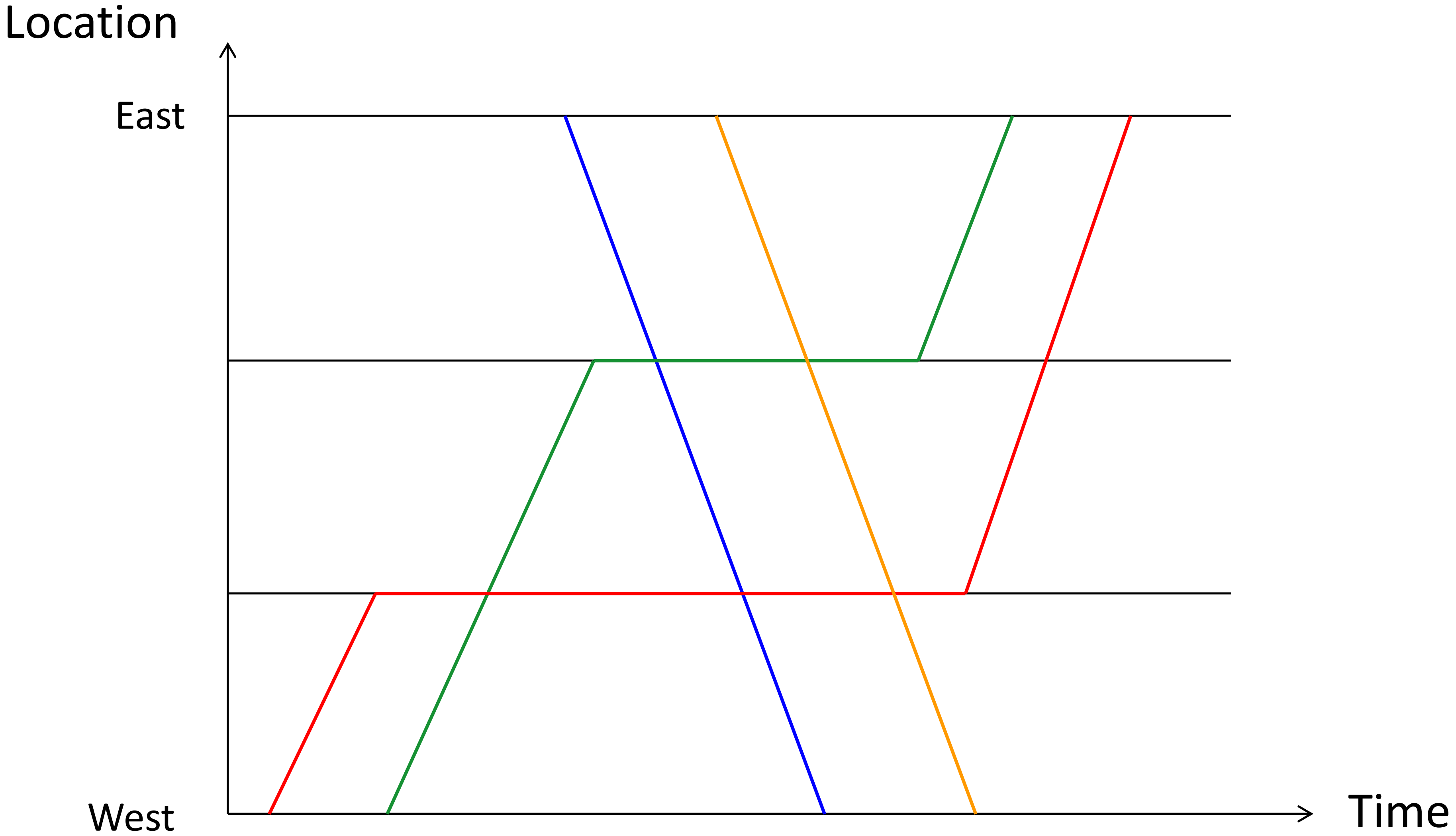
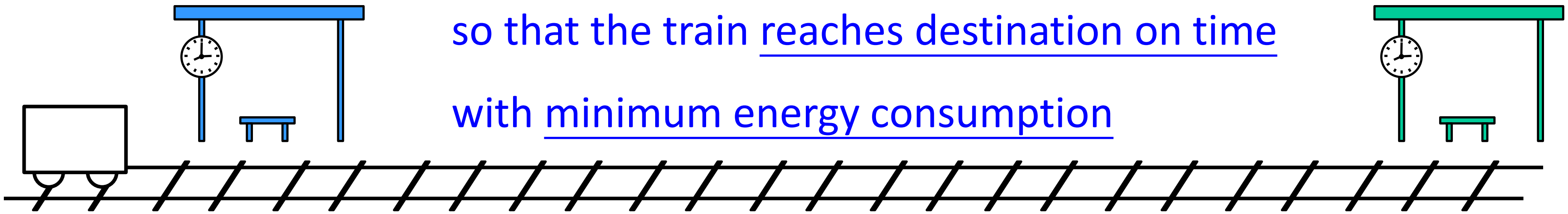
Outline

- Classic energy-efficient train control problems
  - Problems, Formulation, Solution Methods
- Complex train control problems
  - Problems
  - Our proposed multiphase optimal control method
- Case studies
  - Example illustrations
- On-going practical implementation

Schedule

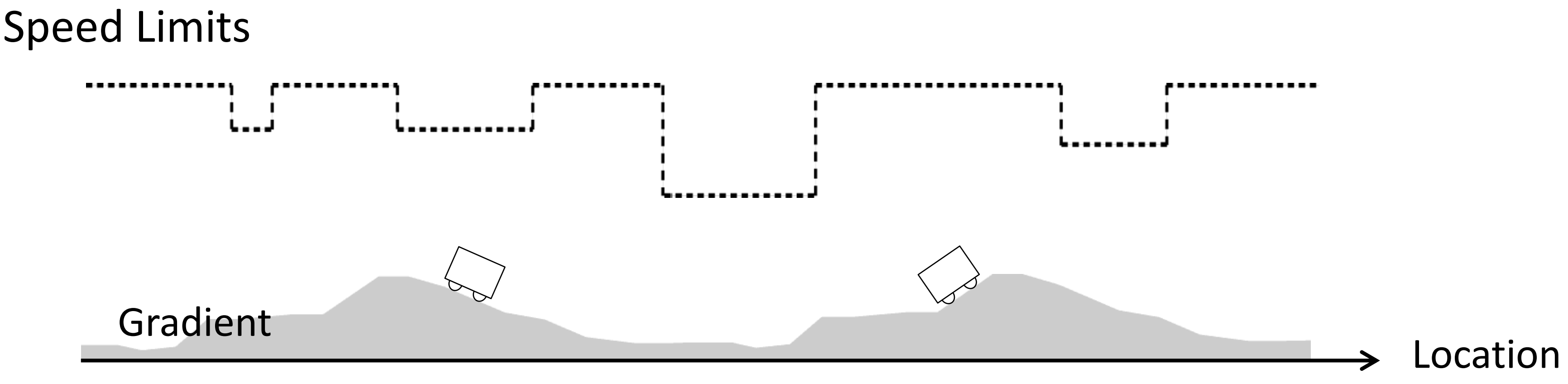


Energy-Efficient Optimal Train Control (aka Speed Profile Optimisation)

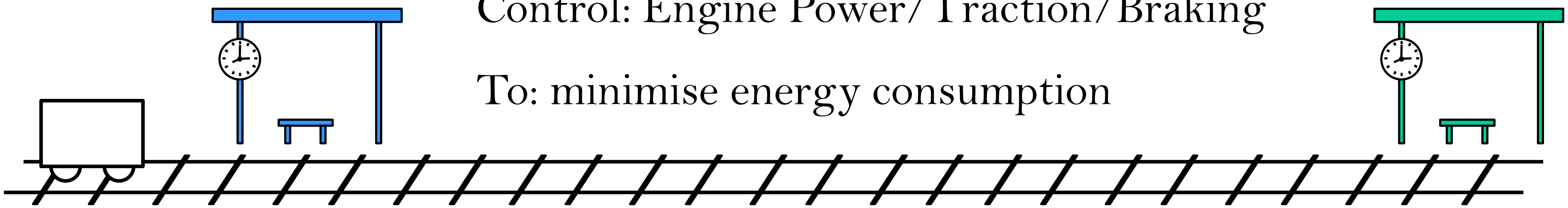


Determine the train running speed profiles so that the train reaches destination on time with minimum energy consumption

Track condition



Optimal train control



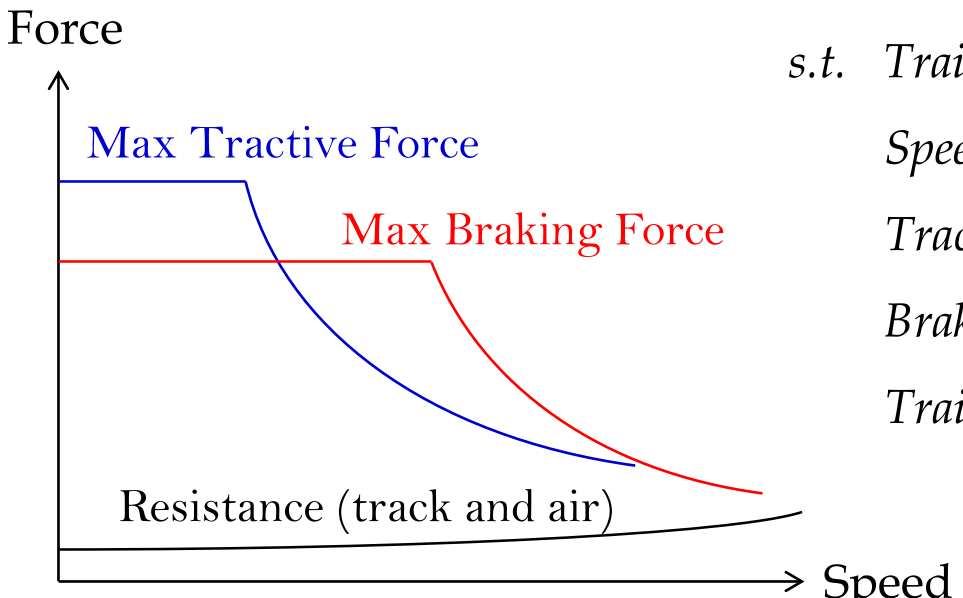
Depart: 8 a.m.

Arrive: 9 a.m.

Control: Engine Power/Traction/Braking

To: minimise energy consumption

Train condition



Minimise *Energy consumption*

s.t. *Train running dynamics (location, speed, forces)*

Speed <= Speed limit

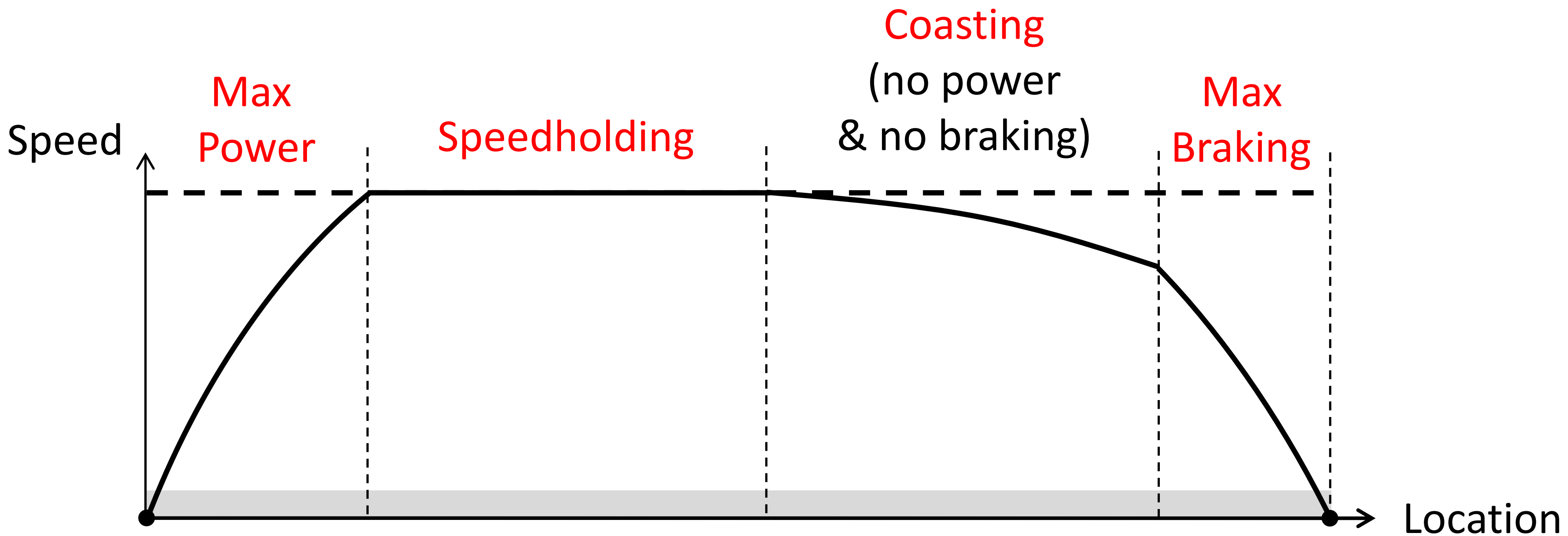
Tractive force <= Maximum Tractive Force

Braking force <= Maximum Braking Force

Train status at origin and destination

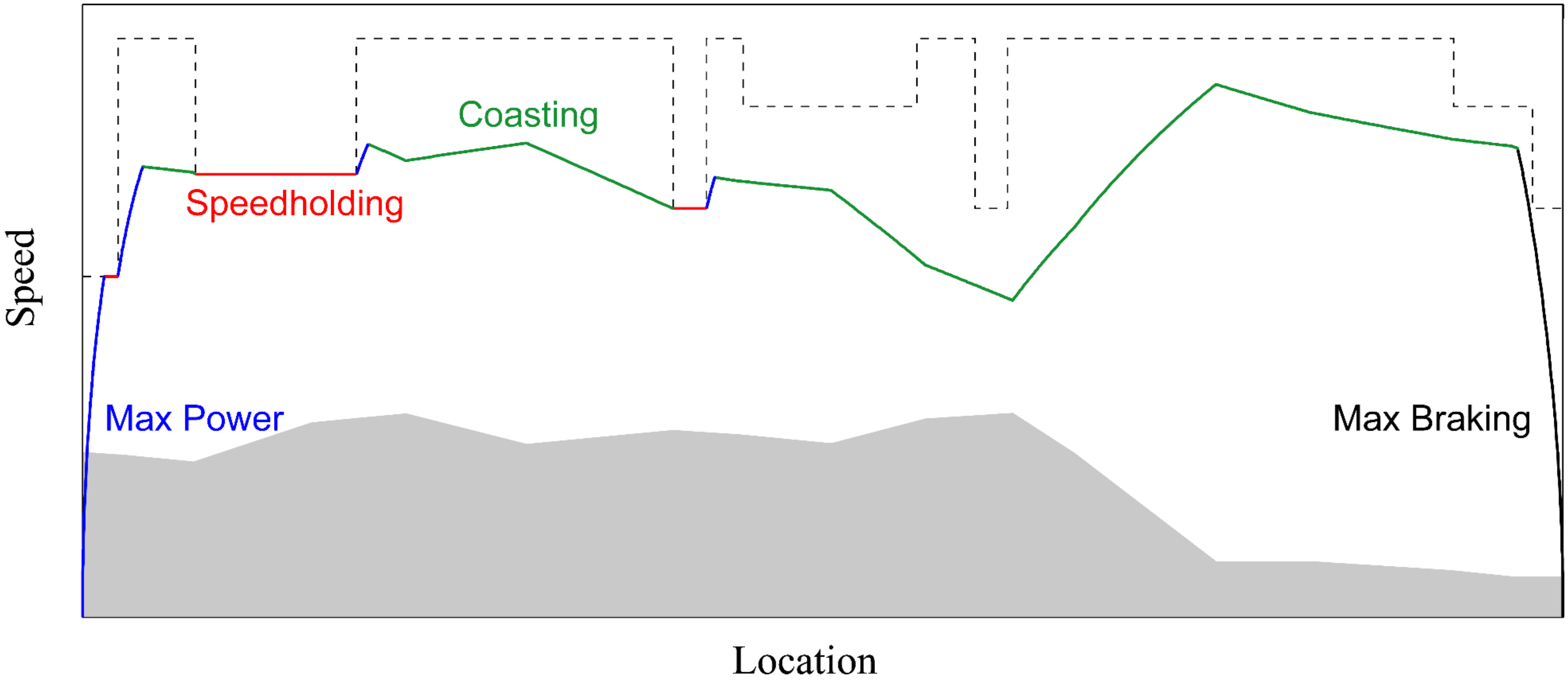
What a Energy-Efficient Speed Profile Looks Like?

- Constant speed limit and constant gradient



What a Energy-Efficient Speed Profile Looks Like?

- Variable speed limit and variable gradient



Formulation of optimal single-train control

$$\min \int_{t_0}^{t_f} \max\{F(t), 0\} v(t)\, \mathrm{d}t$$

s.t. train movement constraint

$$\frac{\mathrm{d}x(t)}{\mathrm{d}t} = v(t)$$

$$\frac{\mathrm{d}v(t)}{\mathrm{d}t} = \frac{1}{M}\big(F(t) - R(v) - G(x)\big)$$

Speed limit constraint

$$0 \le v \le \bar{v}(x)$$

Max forces constraint

$$-\bar{B}(v) \le F \le \bar{F}(v)$$

Boundary constraints

$$x(t_0) = x_0,\ v(t_0) = v_0$$

$$x(t_f) = x_f,\ v(t_f) = v_f$$

$F(t)$: instantaneous applied force (braking/traction)

$v(t)$: speed

$x(t)$: location

M : mass

$R(v)$: running resistance, $= av^2 + bv + c$

$G(x)$: force caused by the track gradient, $= Mg\sin\theta(x)$

$\theta(x)$: track gradient

$\bar{v}(x)$: speed limit

$\bar{F}(v)$: maximum tractive force

$\bar{B}(v)$: maximum braking force

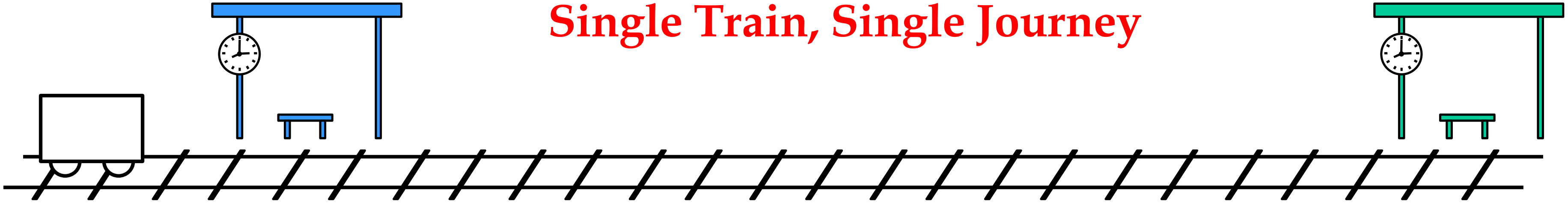
t_0, x_0, v_0: time, location, speed at origin

t_f, x_f, v_f: time, location, speed at destination

Solution Methods

- Optimal control theory - Pontryagin's maximum principle
  - Efficient & good-quality solution, but not applicable for complex problems
- Discretisation and mathematical programming
  - Energy consumption: integral → summation
  - Train movement dynamics: differential equations → difference equations
  - Accuracy is a concern (in the sense of estimating speed and energy consumption), & possible fluctuating solution
- Coasting control
  - Assume coasting once or multiple times on the journey
  - Determine starting time and duration of each coasting
  - Solved by heuristics + simulation
  - Easy to implement, but optimality not guaranteed

Existing solutions



Depart: 8 a.m.

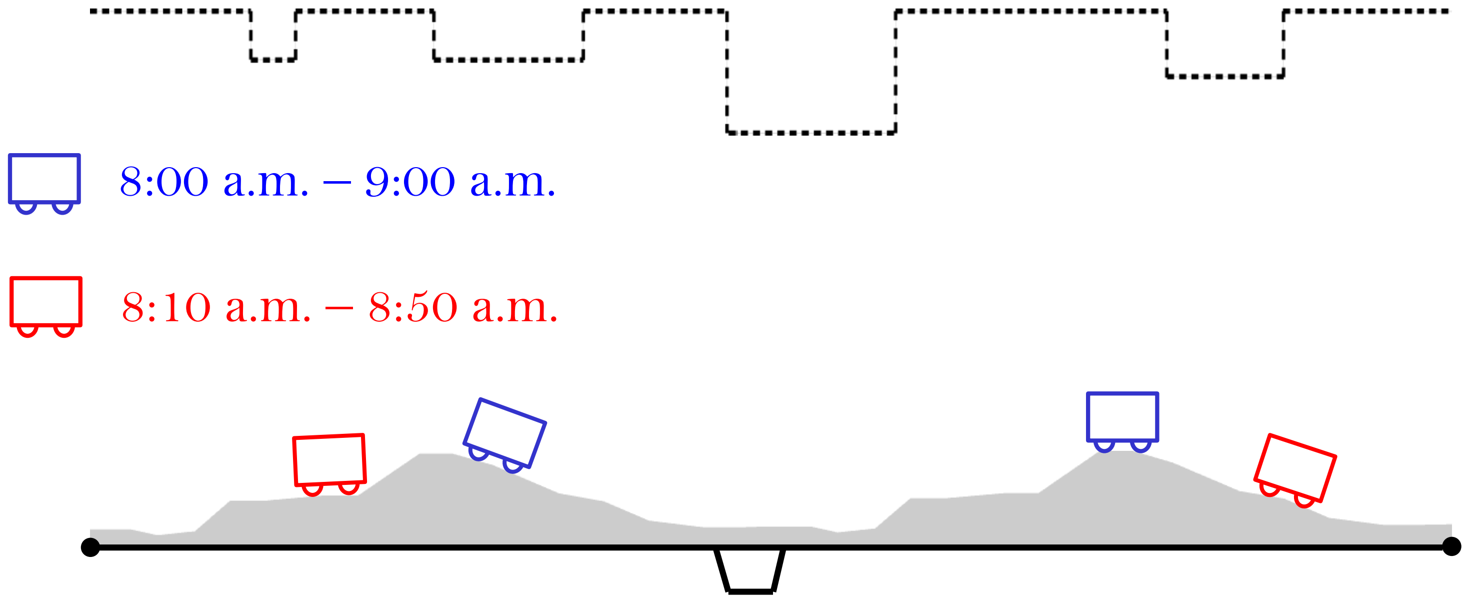
Arrive: 9 a.m.

Single Train, Single Journey

New Problems

- Train control with intermediate constraints
  - e.g. Passing particular track location(s) within particular time/speed window(s)
- Trains following one another, with safety time/distance headway constraints

New Problems: An Example



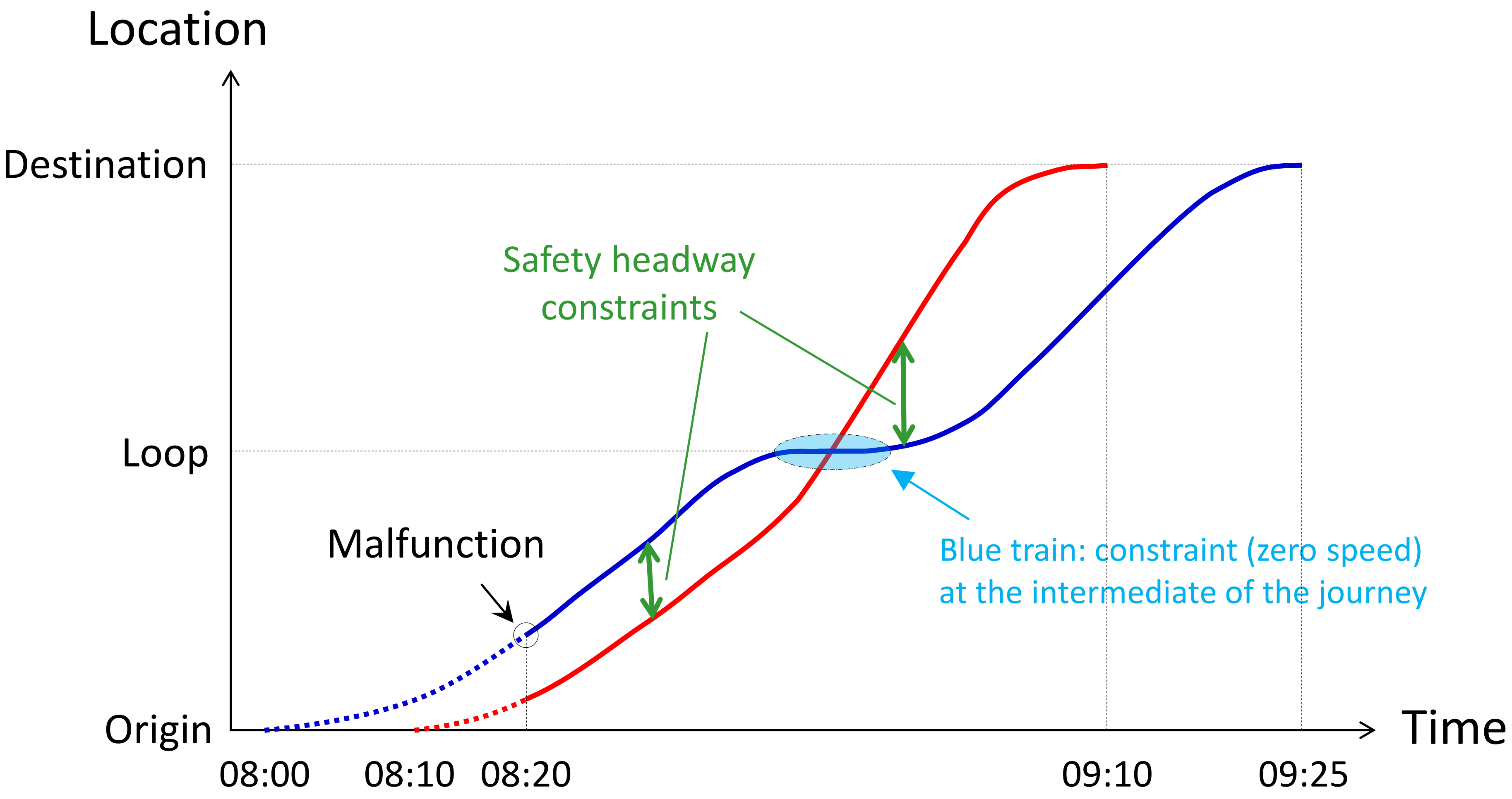
8:00 a.m. – 9:00 a.m.

8:10 a.m. – 8:50 a.m.

Blue train has to stop at the loop to let red train overtake.

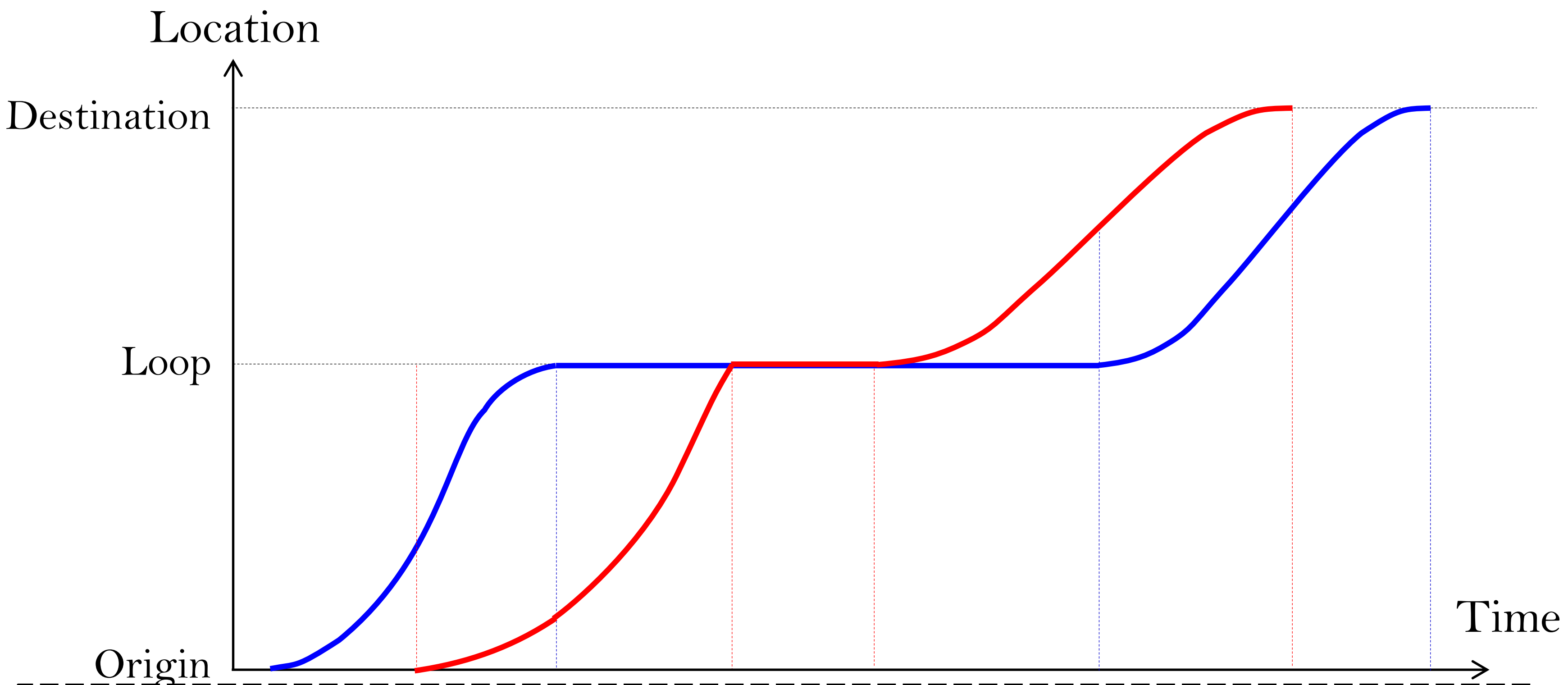
- Blue train’s arrival & departure time at the loop
- Optimal control strategy to drive both trains to the destination

New Problems: An Example



** Ye H; Liu R (2016) A multiphase optimal control method for multi-train control and scheduling on railway lines. Transportation Research Part B: Methodological, 93, pp.377-393.*

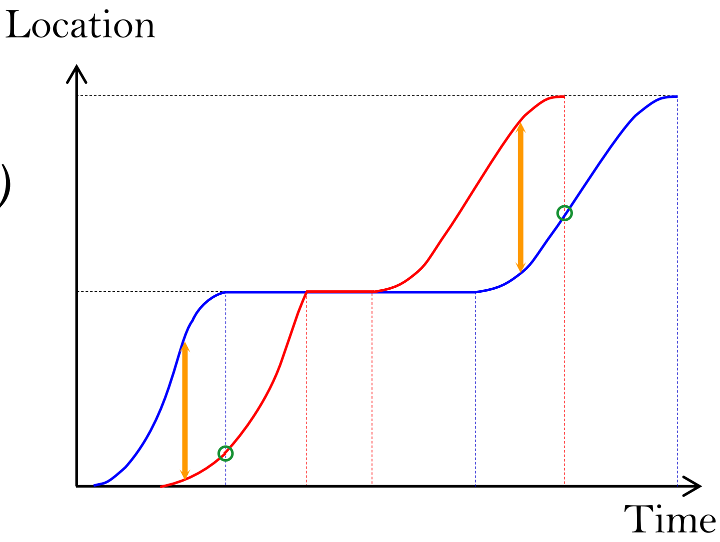
Multiphase optimal control framework



Predetermined	Blue	Red	B	R	R	B	R	B
Event	De	De	Ar	Ar	De	De	Ar	Ar
	S1	S1	M	M	M	M	S2	S2

Phase P1 P2 P3 P4 P5 P6 P7

Multiphase optimal control framework



Minimise *Energy consumption*

s.t. Train running dynamics (location, speed, forces)

Speed <= Speed Limit

Tractive Force <= Maximum Tractive Force

Braking force <= Maximum Braking Force

Train status at beginning and end of each phase

Safety distance between leading and following trains (if any)

Linkage condition on location and speed between two consecutive phases

Additional constraints (if any):

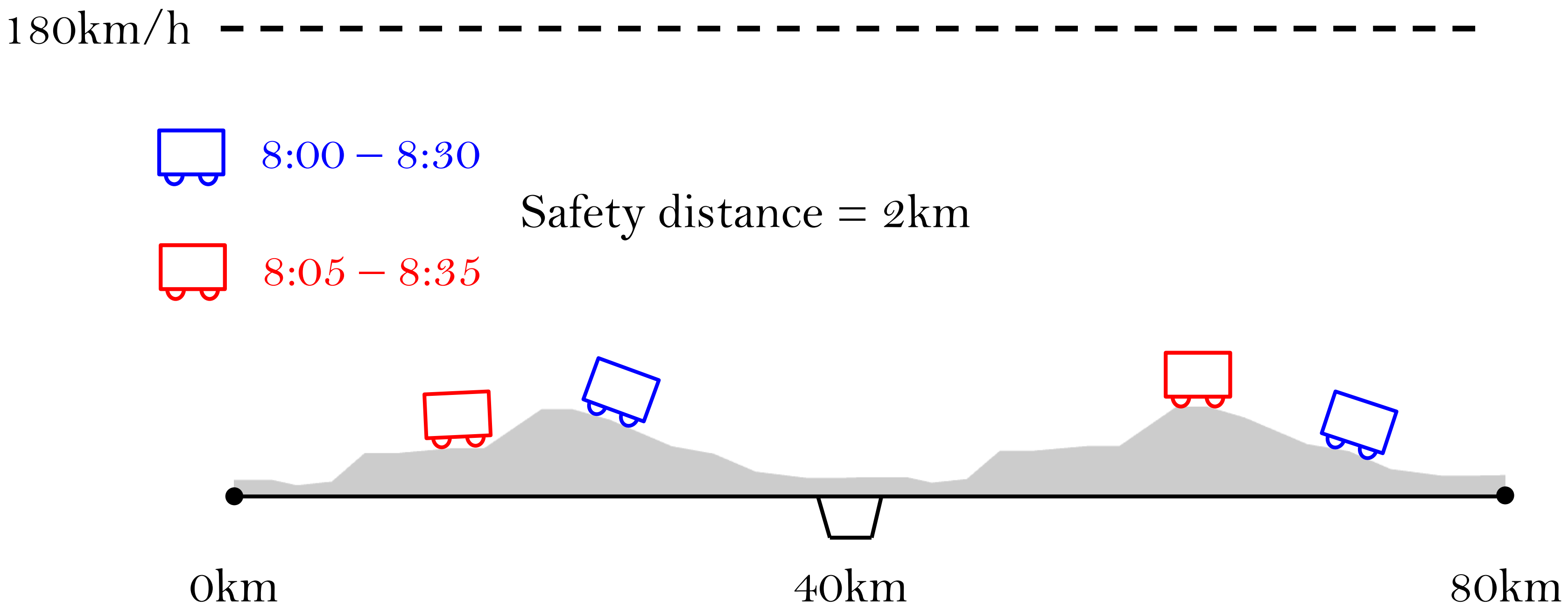
  time window for departure/arrival time at a station

  minimum and maximum stopping times at a station

  maximum interstation running time

- Solved through discretisation, by the pseudospectral method (*GPOPS*)

Case Study 1 - with passing time constraint



180km/h

8:00 – 8:30

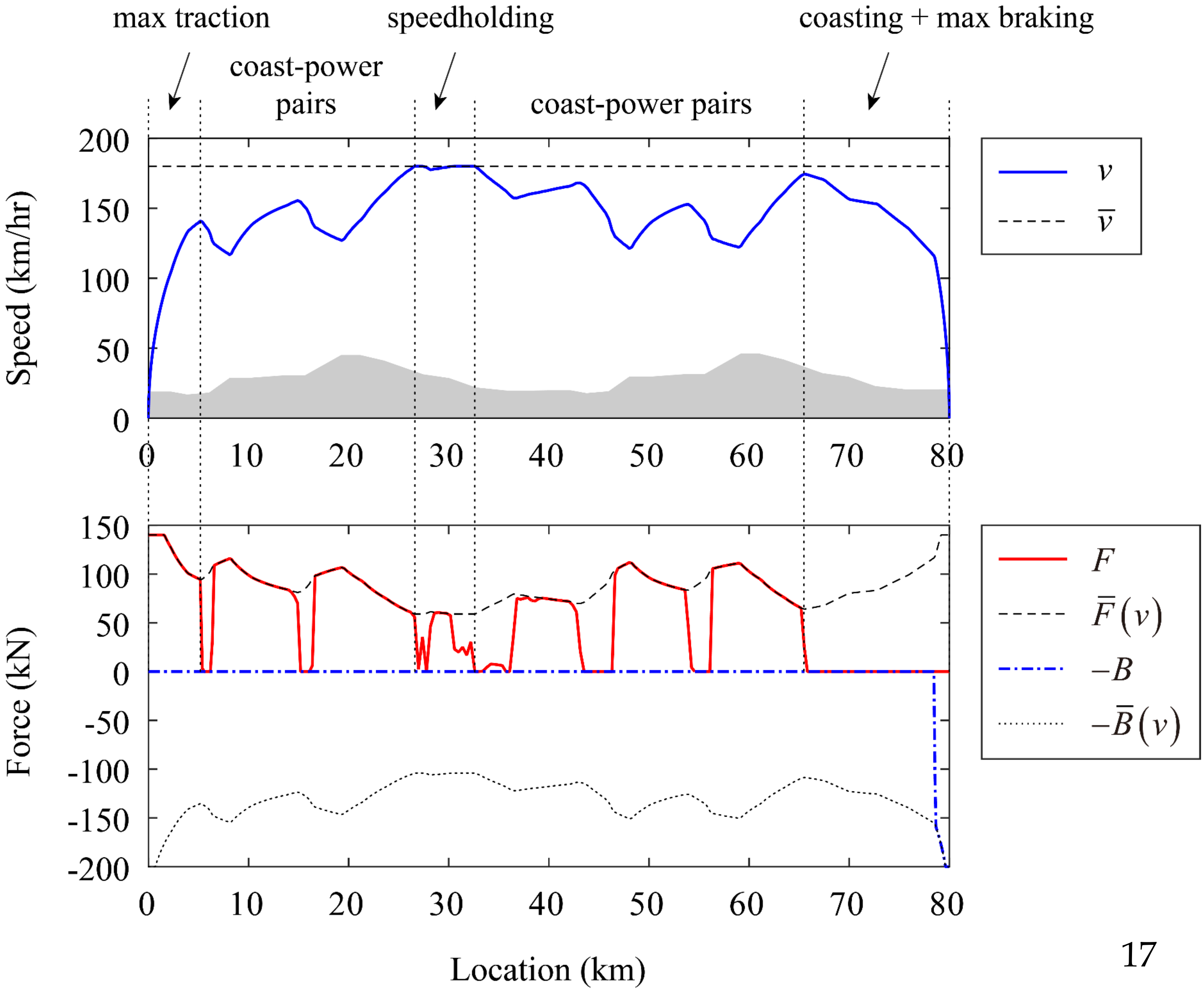
8:05 – 8:35

Safety distance = 2km

0km

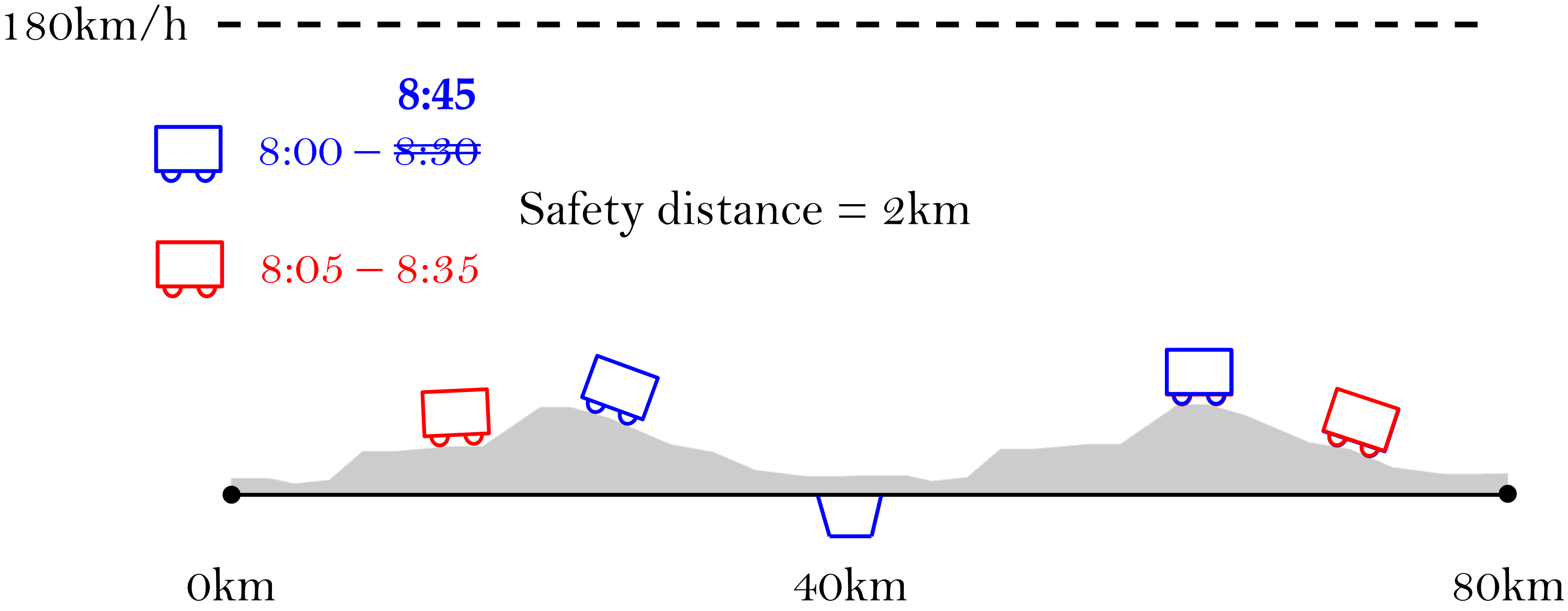
40km

80km


max traction
coast-power pairs
speedholding
coast-power pairs
coasting + max braking
Speed (km/hr)
200
150
100
50
0
0
10
20
30
40
50
60
70
80
v
$\bar{v}$
Force (kN)
150
100
50
0
-50
-100
-150
-200
0
10
20
30
40
50
60
70
80
F
$\bar{F}(v)$
$-B$
$-\bar{B}(v)$
Location (km)

Case Study 2 – with malfunction

A malfunction happens on blue train at 8:10, and its maximum tractive force is reduced



8:45

8:00 – ~~8:30~~

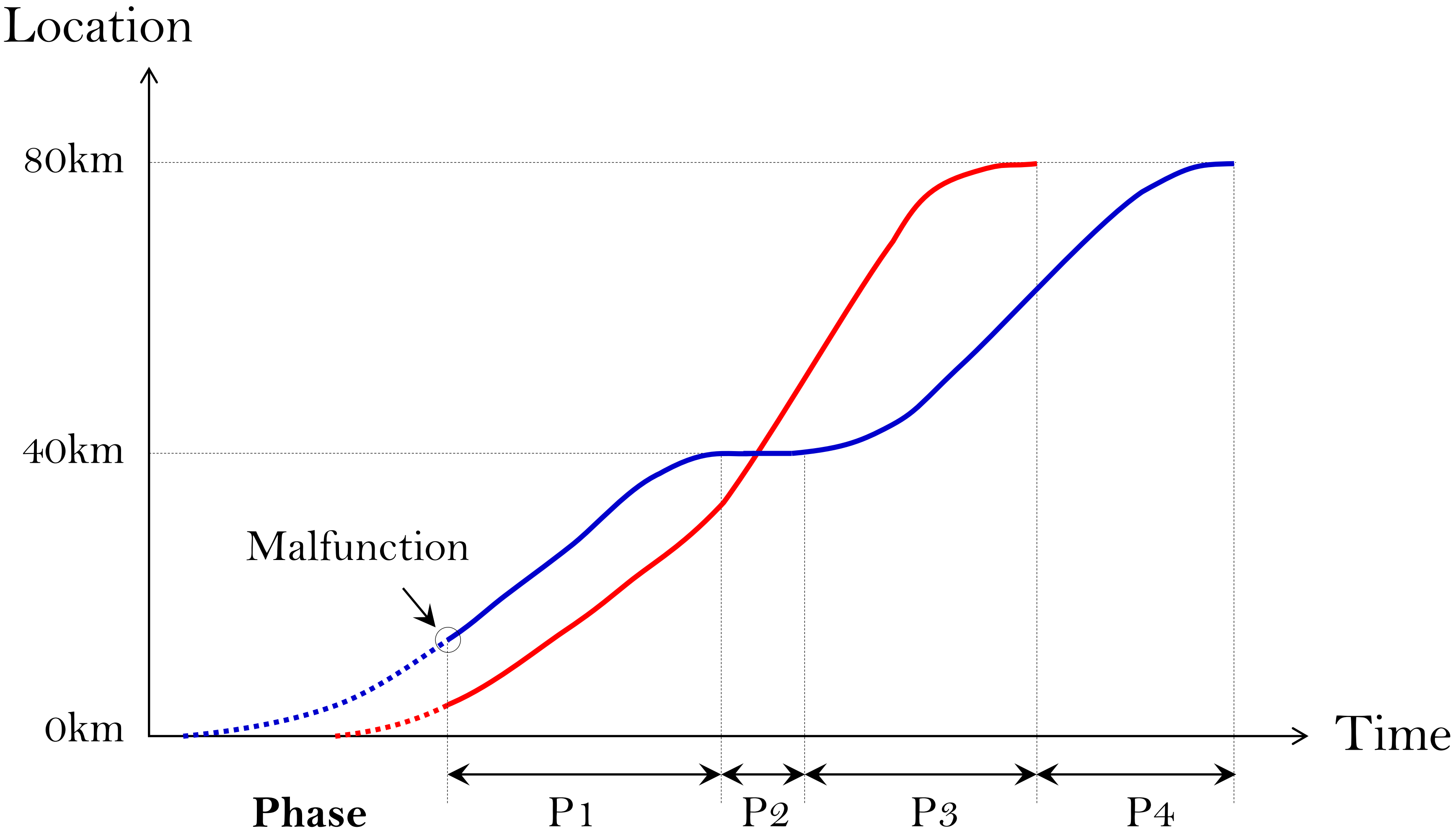
Safety distance = 2km

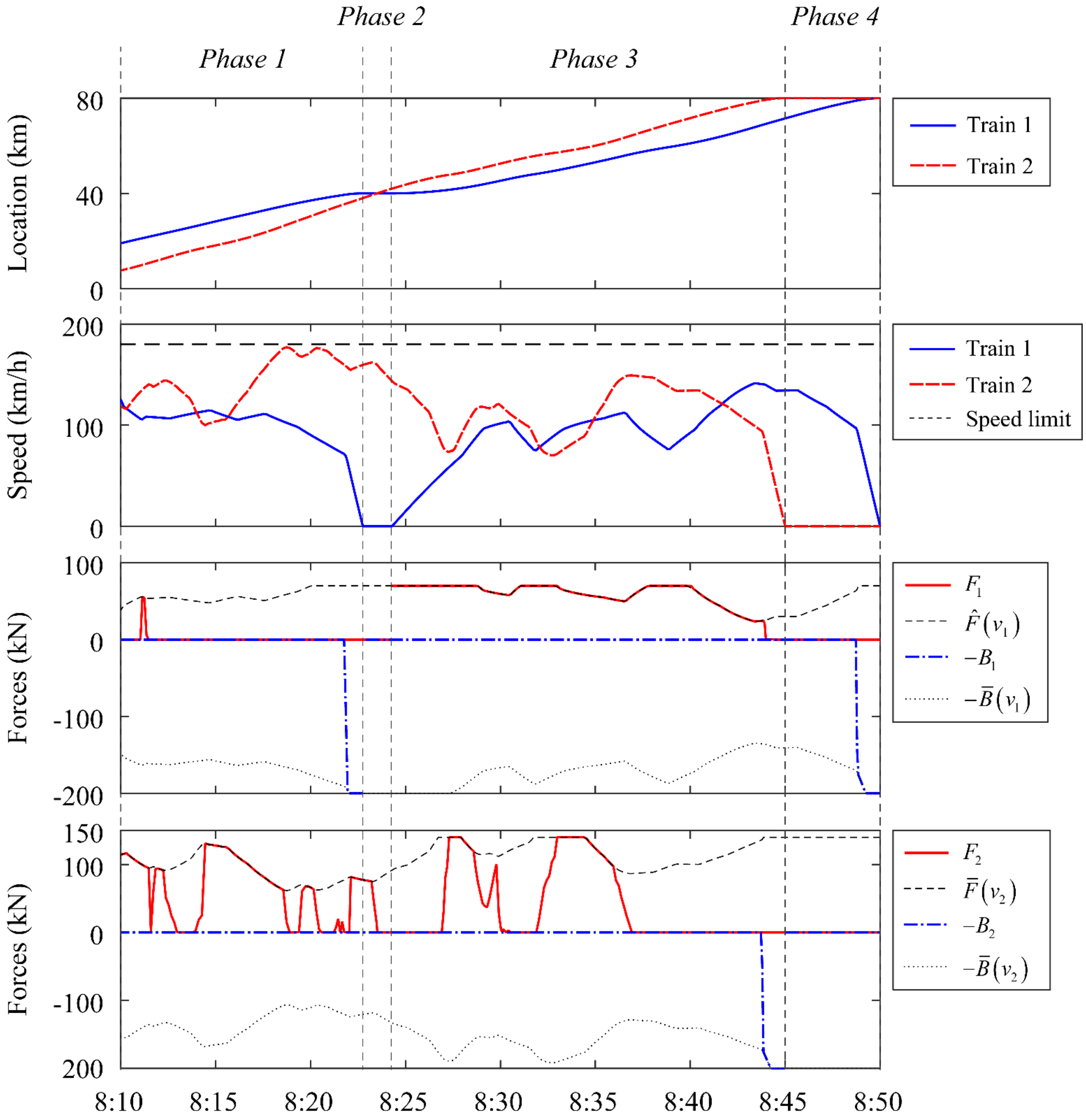
8:05 – 8:35

Blue train has to stop at the loop to let red train overtake.

- Blue train's arrival & departure time at the loop
- Optimal control strategy to drive both trains to the destination

Case Study 2

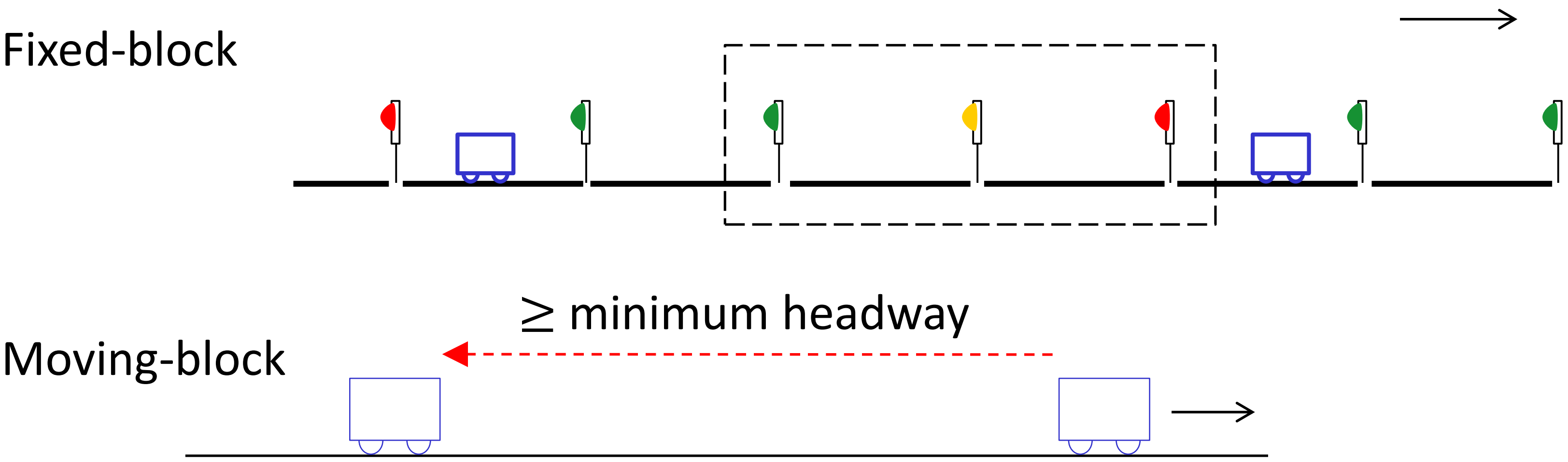




Phase 1
Phase 2
Phase 3
Phase 4
Location (km)
80
40
0
Train 1
Train 2
Speed (km/h)
200
100
0
Train 1
Train 2
Speed limit
Forces (kN)
100
0
-100
-200
F_1
$\hat{F}(v_1)$
$-B_1$
$-\bar{B}(v_1)$
Forces (kN)
150
100
0
-100
200
F_2
$\bar{F}(v_2)$
$-B_2$
$-\bar{B}(v_2)$
8:10
8:15
8:20
8:25
8:30
8:35
8:40
8:45
8:50

New Problems: Leader-follower train pairs

- Train having intermediate constraints: train need to pass particular track location(s) within particular time/speed window(s)
- Several trains following one another, with safety time/distance headway constraints



Case study 3: Leader-Follower Train-pair Control

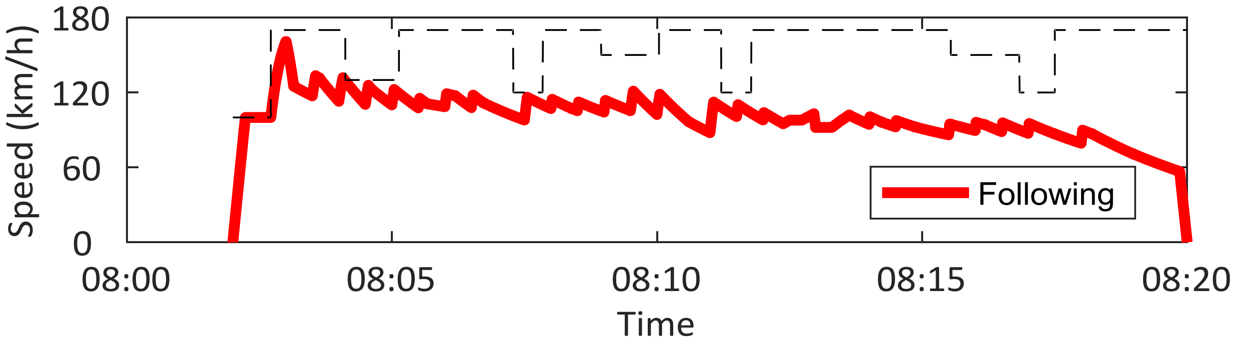
Following: fast train

Leading: slow train

Controlled **Separately**

L: 2885kJ/kg

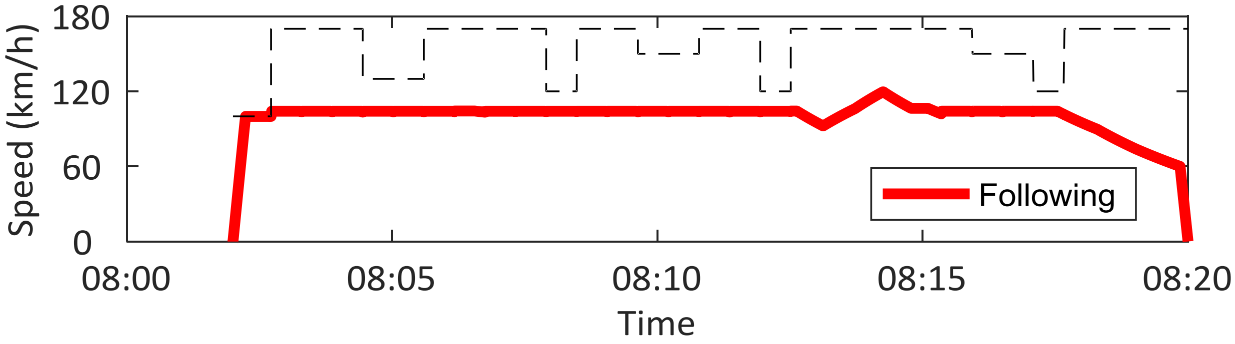
F: 3507kJ/kg



Controlled **Together**

L: 2885kJ/kg

F: 2864kJ/kg **(-18%)**



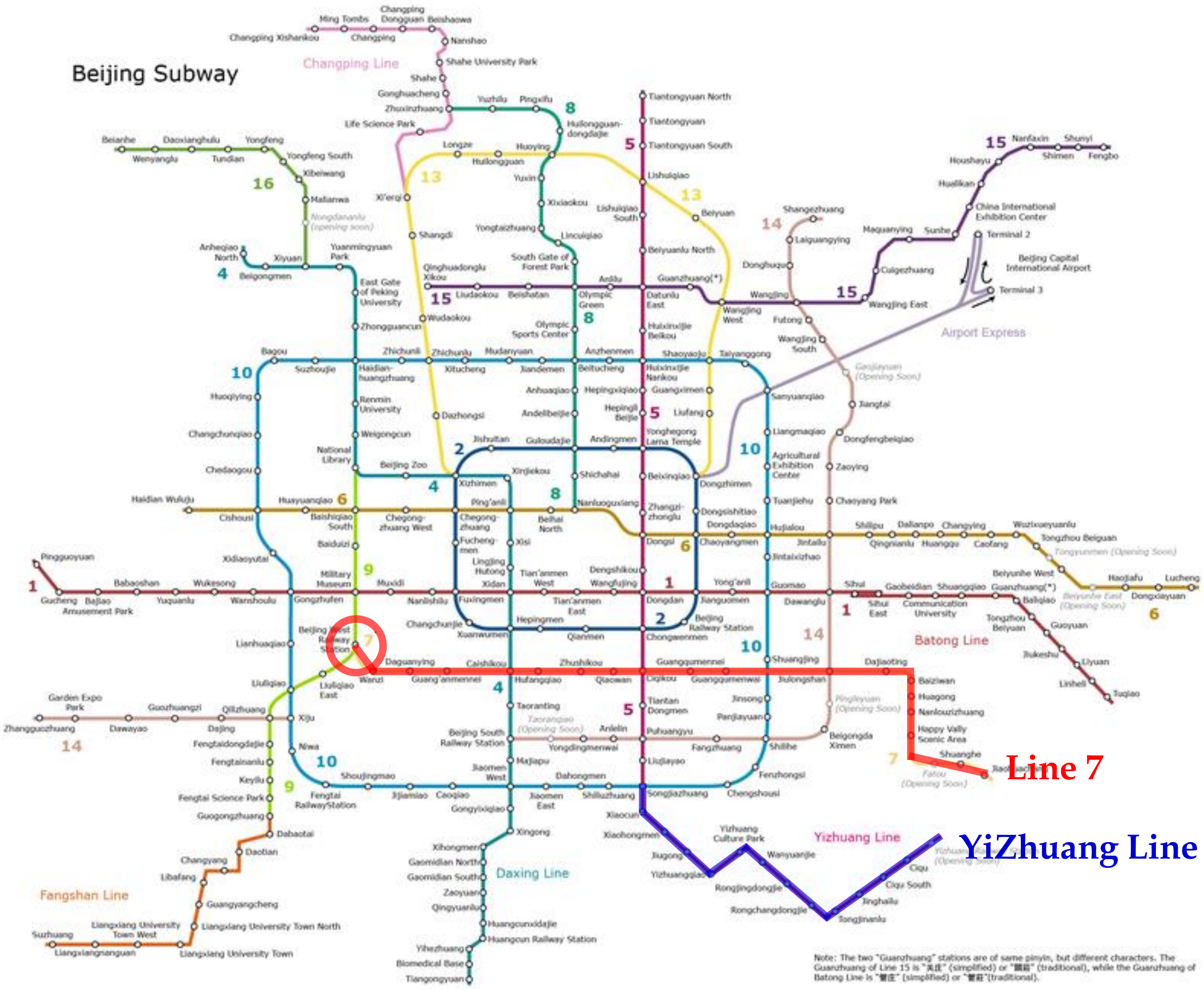
Practical Implementation: Energy-Efficient Automatic Train Operations

- An on-going research project funded by the UK Royal Academy of Engineering
- Project partners:
  - Beijing Jiaotong University (BJTU)
  - University of Leeds (UoL)
  - Traffic Control Technology Ltd (TCT) – signal controller
  - First Group, UK - operator
  - Network Rail, GB – infrastructure manager

Project Overview

- Overall aim: to develop energy-efficient real-time operational ATO based on sound optimal control theory, software verification and real-life test results
- Specific Objectives:
  - To develop efficient train speed profile generation algorithms, based on train-specific characteristics and track conditions, and in real-time
  - To test in laboratory conditions the operations of the algorithms
  - To implement and test in a real-life subway line in Beijing
  - To upscale the R&D impact of the project to other cities in China and to share the experience with UK industry

Beijing Subway Network



Line 7

YiZhuang Line

Challenges and Opportunities

- Would be the first in the world to deploy an optimal train-speed-profile (TSP) on ATO in real-life train operations
- Fast computation to re-calculate TSP in real-time
- Scope for co-optimising TSP & schedules (for Line 7)
- Upscaling impact potentials in China
  - 27 cities with metro networks
  - 15 others building/planning to build metro lines

References

- Ye, H., Liu, R., 2016. A multiphase optimal control method for multi-train control and scheduling on railway lines. Transportation Research Part B 93, 377-393.
- Ye, H., Liu, R., 2017. Nonlinear programming methods based on closed-form expressions for optimal train control. Transportation Research Part C 82, 102-123.

Thank You

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